AE2. **Devoir n° 2 : dynamique des fluides - Éléments de correction**

Ex1 1.1. Equation de continuité : $\boxed{Q_v = S \times v = \text{constante}}$

1.2. Entre les points 1 et 2, on peut écrire : $S_1 v_1 = S_2 v_2$

$$d_2 = \frac{d_1}{2} \Rightarrow S_2 = \frac{\pi d_2^2}{4} = \frac{\pi}{4}\left(\frac{d_1}{2}\right)^2 = \frac{\pi}{4}\frac{d_1^2}{4} = \frac{S_1}{4}$$

d'où $v_2 = \frac{S_1}{S_2} v_1 = 4 v_1 = 4 \times 20 = \boxed{80 \text{ cm}\cdot\text{s}^{-1}}$

ou $S_1 = \frac{\pi d_1^2}{4} = \pi \times \frac{10^2}{4} = 78{,}5 \text{ cm}^2$

$S_2 = \frac{\pi d_2^2}{4} = \pi \times \frac{5^2}{4} = 19{,}6 \text{ cm}^2$

$v_2 = \frac{S_1 v_1}{S_2} = \frac{78{,}5}{19{,}6} \times 20 = 80 \text{ cm}\cdot\text{s}^{-1}$

1.3. A la sortie de l'évent : $d_3 = d_1 \Rightarrow S_3 = S_1$ et donc $v_3 = \frac{S_1}{S_3} v_1 = v_1 = \boxed{20 \text{ cm}\cdot\text{s}^{-1}}$

2.1. Equation de Bernoulli entre l'intérieur de l'évent (2) et la sortie (3) :

$$\boxed{P_2 + \rho g z_2 + \frac{1}{2}\rho v_2^2 = P_3 + \rho g z_3 + \frac{1}{2}\rho v_3^2}$$

2.2. L'évent est une ouverture horizontale : $z_3 = z_2$

Point 2 $\begin{cases} z_2 \\ v_2 = 80 \text{ cm}\cdot\text{s}^{-1} = 4 v_1 \\ P_2 \end{cases}$ Point 3 $\begin{cases} z_3 = z_2 \\ v_3 = v_1 \\ P_1 \end{cases}$

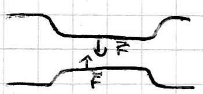
D'après le théorème de Bernoulli : $P_2 + \cancel{\rho g z_2} + \frac{1}{2}\rho v_2^2 = P_3 + \cancel{\rho g z_3} + \frac{1}{2}\rho v_3^2$

$$\Rightarrow P_2 - P_3 = \frac{1}{2}\rho v_3^2 - \frac{1}{2}\rho v_2^2 = \frac{1}{2}\rho\left[v_3^2 - v_2^2\right] = \frac{1}{2}\rho\left[v_1^2 - \underbrace{(4v_1)^2}_{16 v_1^2}\right]$$

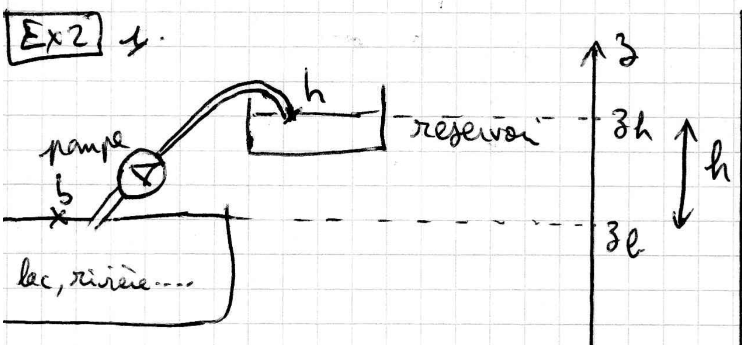
$$\boxed{P_2 - P_3 = -\frac{15}{2}\rho v_1^2}$$

2.3. $P_2 - P_3 = -\frac{15}{2} \times 1{,}2 \times (0{,}2)^2 = \boxed{-0{,}36 \text{ Pa}}$

2.4. $F = (P_2 - P_3) \times A = -0{,}36 \times (750 \times 10^{-4}) = \boxed{-0{,}027 \text{ N}}$



Ex2 1.



2/ 2.1. $Q_v = \frac{\Delta V}{\Delta t} = \frac{V_e}{\theta} = \frac{100}{1} = 100 \text{ m}^3/\text{h}$

$Q_v = \frac{100}{3600} \text{ m}^3/\text{s} = \boxed{27{,}8 \cdot 10^{-3} \text{ m}^3\cdot\text{s}^{-1}}$

2.2. $S = \frac{\pi D^2}{4} = \pi \times \frac{175^2}{4} = 24100 \text{ mm}^2 = \boxed{24{,}1 \times 10^{-3} \text{ m}^2}$

2.3. $Q_v = v_{eau} \times S \Rightarrow v_{eau} = \frac{Q_v}{S} = \boxed{1{,}15 \text{ m}\cdot\text{s}^{-1}}$

3.1. $Q_m = \rho Q_v = 1000 \times 27{,}8 \cdot 10^{-3} = \boxed{27{,}8 \text{ kg}\cdot\text{s}^{-1}}$

3.2. On applique le th. de Bernoulli généralisé pour un écoulement de (b) vers (h) :

$$\frac{1}{2}\rho(\underbrace{v_h^2}_{= v_{eau}} - \underbrace{\cancel{v_b^2}}_{=0}) + \rho g \underbrace{(z_h - z_b)}_{h} + P_h - P_b = \frac{P_{up}}{Q_v}$$

$$\Rightarrow \boxed{P_{up} = Q_v\left[\frac{1}{2}\rho v_{eau}^2 + \rho g h + P_h - P_b\right]}$$

$P_{up} = 27{,}8 \cdot 10^{-3} \times \left[\frac{1000 \times (1{,}15^2)}{2} + 1000 \times 10 \times 40 + 1 \cdot 10^5 - 1{,}05 \cdot 10^5\right] = 11{,}0 \cdot 10^3 \text{ W} = \boxed{11{,}0 \text{ kW}}$

3.3. $P_{up} = Q_m \times W \Rightarrow W = \frac{P_{up}}{Q_m} = \boxed{396 \text{ J}\cdot\text{kg}^{-1}}$

Ex2 (suite)

3.4. $\eta = \frac{P_{up}}{P_{élec}}$

$\Rightarrow P_{élec} = \frac{P_{up}}{\eta} = \frac{11}{0{,}7} = \boxed{15{,}7 \text{ kW}}$